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HYDRODYNAMIC PROBLEMS IN THREE DIMENSIONS

By P. G. Hubbard, J. M. ASCE, and S. C. Ling

HYDRAULICS DIVISION

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PAPERS

HYDRODYNAMIC PROBLEMS IN THREE DIMENSIONS

By P. G. Hubbard,[1] J. M. ASCE, and S. C. Ling[2]

Synopsis

The rather familiar analog utilizing an electrolytic bath to represent potential fluid motion has been refined and throughly tested in connection with fundamental investigations of fluid flow. Principles and techniques of the investigations are fully presented in this paper with emphasis on the general approach for three-dimensional flow and typical results. Solution of a representative problem is given.

Introduction

The particular electrical analogy with which this paper is concerned fits within the rather broad classification frequently referred to as the "electrolytic-tank method." In common with several other analogs, it represents potential motion in the prototype; that is, motion that may be described by a velocity potential with respect to the axes of motion $\phi\ (x, y, z)$ such that

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The motion also possesses the property that the gradient of the scalar field described by ϕ is the velocity vector,

$$v_x = \frac{\partial\phi}{\partial x};\ v_y = \frac{\partial\phi}{\partial y};\ v_z = \frac{\partial\phi}{\partial z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which v is the velocity. Electric field theory includes a similar concept for voltage E, with a scalar field $E(x, y, z)$ that also satisfies the Laplace equation:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Note.—Written comments are invited for publication; the last discussion should be submitted by February 1, 1953.

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and whose gradient is the vector representing ϵ, the electric field strength,

$$\epsilon_x = \frac{\partial E}{\partial x}; \quad \epsilon_y = \frac{\partial E}{\partial y}; \quad \epsilon_z = \frac{\partial E}{\partial z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

This mathematical similarity forms the basis for the analogy, and the application of the electrolytic-tank method involves the exploration of a conductor geometrically similar to the corresponding flow system. The application of a voltage $(E_1 - E_2)$ across the conductor, between zones representing the entrance and exit of the fluid in the system, establishes within the conductor a potential field similar to that of its fluid prototype. The potential pattern is obtained by means of electrodes in contact with the conductor, using standard electrical potentiometer systems. The details of the particular method described in this paper have been presented previously.[3]

Experimental Techniques

The conductor representing the flow system in the analog is usually an electrolytic solution, and it is held to the proper form by nonconductors, representing flow boundaries (or planes of flow symmetry), and by metal anodes representing equipotential surfaces at the extremities of the system. In constructing the plastic forms representing flow boundaries, extreme care must be exercised to hold tolerances within close limits, especially at points of curvature at which the pressure gradients would be relatively great. The models should be made on as large a scale as is convenient, of course, but the size limitations usually encountered cause the errors from small deviations to be greatly magnified. For the same reason, the electrodes used to measure potential differences should be quite small in area, and their spacing should be measured with high precision. Fixed electrodes flush with the surface of the models can usually be located with greater accuracy than a movable probe, and fixed electrodes also present less disturbance to the flow. Since the most extreme conditions must occur at the boundary, the absence of any measurement in the fluid interior is not important.

The electrical potentiometers employed should be capable of measuring potential differences with a precision at least equal to that with which electrode spacings are measured. The potentiometers used in all tests at the Iowa Institute of Hydraulic Research, State University of Iowa, in Iowa City, measured the potential at each point with an accuracy of 0.01% of the total applied potential. The only instruments needed were a four-dial decade potentiometer and a sensitive vacuum-tube millivoltmeter. The latter was necessary because of the low applied potential (6 volts) and the need for a high-impedance null detector. The millivoltmeter internal impedance of 5 megohms (see Fig. 1) assures negligibly small currents through the electrodes even when the system is far from balance, thus avoiding distortion of the potential pattern. The sensitivity of 0.0003 volt of the millivoltmeter assures adequate response for the detection of the smallest change on the decade potentiometer. Miniature tubes and battery supply were used to

[3] "Application of the Electrical Analogy in Fluid Mechanics Research," by P. G. Hubbard, *Review of Scientific Instruments*, November, 1949.

keep size and capacitance to ground at a minimum. In use, the potentiometer was simply connected in parallel with the power supply to the anodes, while the millivoltmeter was connected between the movable tap on the potentiometer and an electrode. The dials were then adjusted until the volt-

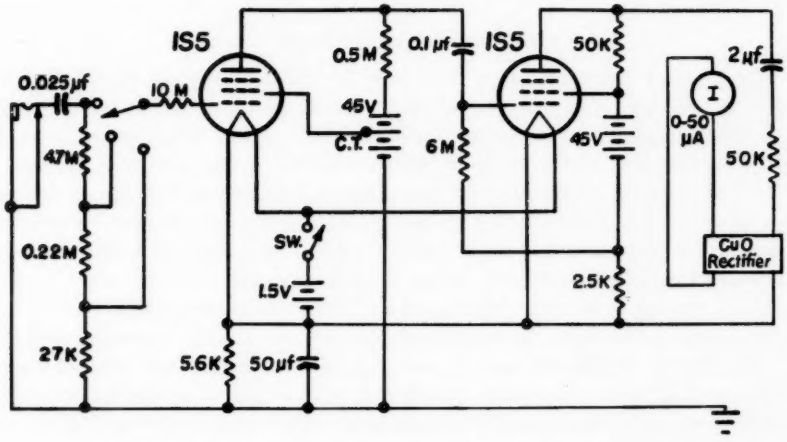

FIG. 1.—VACUUM-TUBE MILLIVOLTMETER USED AS NULL DECTECTOR

age difference was a minimum, using a modified Wagner ground (a second potentiometer with movable tap connected to ground through a 0.05 microfarad condenser) to increase the sharpness of the null if necessary. The readings of the four dials thus gave the relative potential directly, and no other electrical quantity was required.

The direction of the streamlines (except for their tangency to the boundary surface) will not be known in advance in the general case, and this fact must be considered in locating electrodes and computing velocity or pressure distributions. To facilitate the determination of the resultant velocity vector, it is convenient to place electrodes in an orthogonal (x, y) pattern on the developed boundary, with electrode spacings Δx and Δy maintained small relative to the radius of curvature. Then, since the boundary is a stream surface, the resultant velocity v_s may be computed from Δx, Δy, and the corresponding potential difference $\Delta\phi$ between any pair,

$$v_s = \frac{\partial\phi}{\partial s} \doteq \frac{\Delta\phi}{\Delta s} = \sqrt{\left(\frac{\Delta\phi}{\Delta x}\right)^2 + \left(\frac{\Delta\phi}{\Delta y}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The corresponding piezometric head h may be obtained from the Bernoulli relationship:

$$\frac{h - h_0}{\frac{V^2_0}{2g}} = 1 - \left(\frac{v}{V_0}\right)^2 \doteq 1 - \frac{(\Delta\phi/\Delta x)^2 + (\Delta\phi/\Delta y)^2}{(\Delta\phi/\Delta x)^2_0 + (\Delta\phi/\Delta y)^2_0} \quad \ldots\ldots\ldots\ldots (6)$$

The quantities $(\Delta\phi/\Delta x)_0$ and $(\Delta\phi/\Delta y)_0$ are obtained from measurements be-

tween electrodes placed in a region of uniform flow at velocity V_0 and piezometric head h_0. Pressure distributions are obtained by plotting points from Eq. 6 approximately midway between the corresponding electrodes and then adjusting the curves so that the area under the curve is equal to that beneath a bar graph drawn through individual points.

Evaluation of Earlier Tests

The method described was first used at the Iowa Institute of Hydraulic Research in 1945, and has undergone a continuous modification since that time, being used for both research and design in three-dimensional problems. Until the initiation of the tests described in this paper, however, its use was limited to axially symmetrical flow systems because of the desire to check thoroughly and continually on its accuracy before presenting it as a reliable tool. Many of the results were verified either analytically, using the potential concept, or experimentally, using a water tunnel.

These earlier tests,[3,4,5,6] fell naturally into two classifications based on the relationship of the fluid to the associated boundary: (1) Those in which the flow is external to the boundary, such as various head forms for submerged bodies; and (2) those in which the flow is internal such as conduit inlets and contractions or a free liquid jet impinging upon a plane normal to its axis. The nature of these systems demonstrates the basic requirements involved in the use of the analogy, namely, that viscous effects must be negligible and that the flow pattern must be determined solely by the boundary geometry. Thus, decelerating flow around tail forms or through expansions or outlets that involve boundary-layer separation, cannot be represented by this electrical analog. Moreover, the analog may not represent the actual pattern even for generally accelerating flow if regions of local deceleration exist that tend to cause local separation. The analog will indicate the existence of such conditions, however, and one of its most valuable uses is in testing and modifying designs to obtain the most economical separation-free form.

The analogy has proved useful—particularly for extensive exploratory surveys—because of its flexibility, its economy of time compared with analytical methods in general, and its economy of equipment compared with hydrodynamic or aerodynamic models. These virtues will become apparent as the description of a particular series proceeds. In most tests, the accuracy has been sufficient for design purposes, indicating velocities that deviate by less than 1% from those determined by other methods. In one particular test reported earlier,[5] however, an inadequate length of uniform section led to over-optimistic conclusions concerning circular inlets. The error was not discovered until the new series of tests described in this paper was completed, indicating results incompatible with the former. When careful checking and reconstruction of the analog continued to disprove the earlier

[4] "Use of the Three-Dimensional Electrical Analogy in the Design of Conduit Contractions," by M. M. Hassan, dissertation presented to the State University of Iowa, at Iowa City, Iowa, in August, 1948, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[5] "Cavitation-Free Inlets and Contractions," by H. Rouse and M. M. Hassan, *Mechanical Engineering*, March, 1949.

[6] "Deflection of a Liquid Jet by a Perpendicular Boundary," by A. Leclerc, thesis presented to the State University of Iowa, at Iowa City, Iowa, in August, 1948, in partial fulfilment of the requirements for the Degree of Master of Science.

conclusions, a mathematical check, using the relaxation method, was made. The results were in accordance with those presented in the final section of this paper. If proper care is given to all phases of construction, measurement, and interpretation, results of any desired accuracy should be obtainable.

Use of the Analog

Typical Tests.—Having established the accuracy of the method by experiments on several analogs of axially symmetrical flow systems in which a comparison could be made with analytical methods, it appears safe to use the electrical analogy to represent a design problem in which the absence of an axis of symmetry makes a mathematical analysis impractical or virtually impossible. Such problems naturally represent one of the most valuable applications of the electrical analogy since the only alternative method of solution is a series of expensive models. One system of practical importance is that of the transition, using cylindrical sections, from a reservoir of fluid to a conduit of square cross section. This configuration appears frequently in the design of hydraulic inlet works, in which the prediction of low-pressure zones is important because of the danger of cavitation. Since a study to minimize the tendency toward separation in an inlet is well-suited to the capa-

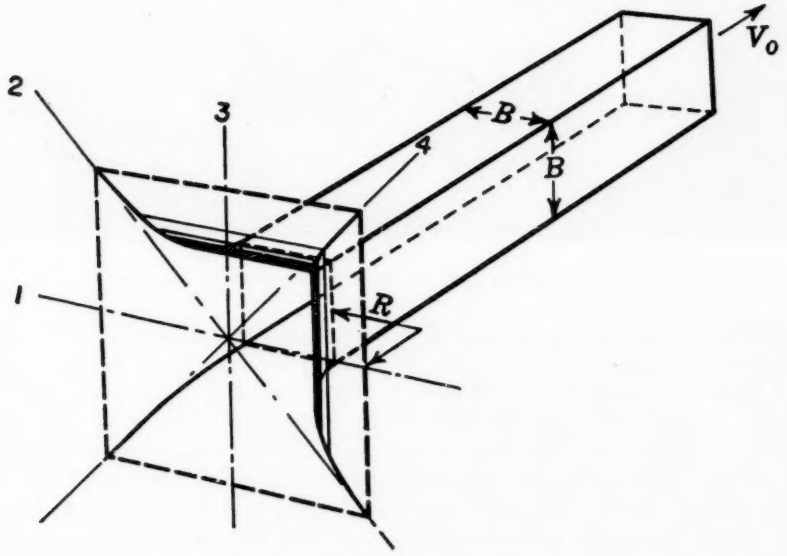

Fig. 2.—Schematic Diagram of Inlet

bilities of the electrical analogy and is of practical importance, it has been chosen to demonstrate the application of this method.

The inlet to a square conduit has four planes of symmetry that divide it into eight segments (Fig. 2). The potential flow pattern in the entire structure may be determined by a study of the pattern in any one of these segments

bounded by two adjacent planes of symmetry and a section of the transition boundary. In the electrical analog (Fig. 3), a sheet of plate glass inclined at an angle of 45° to the horizontal represents a plane passing through the center lines of opposite sides of the conduit, and the sections representing the

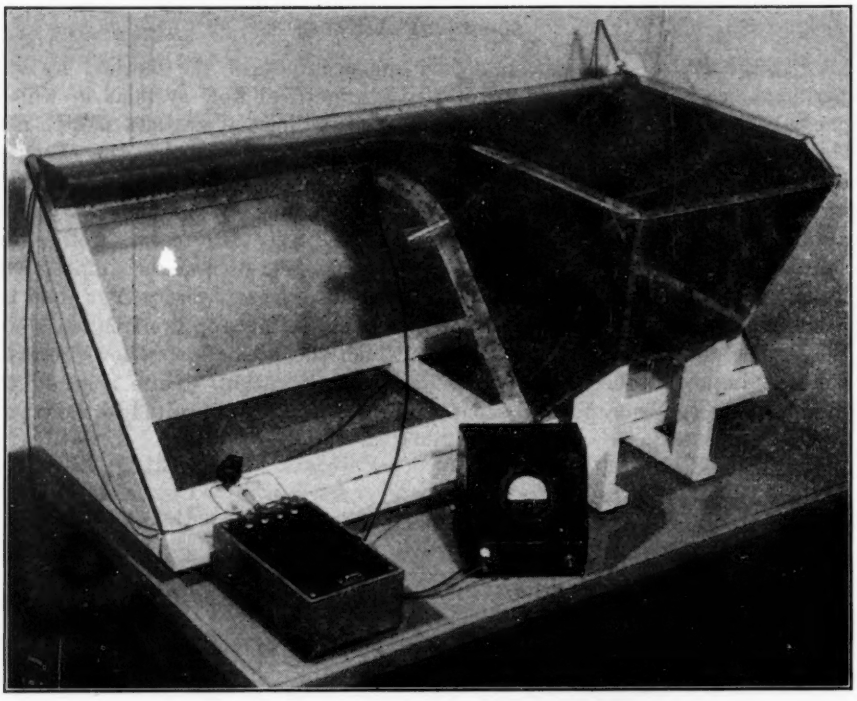

FIG. 3.—ELECTRICAL ANALOG FOR SOLUTION OF INLET PROBLEM

transition boundary are formed of Lucite sheets and a section of a Lucite cylinder. At zones corresponding to equipotential surfaces in the conduit and reservoir, respectively, anodes of sheet copper and copper screen are placed. Small electrodes inserted in an orthogonal pattern in the Lucite sections complete the form without the use of precision machine tools. When the copper sulfate solution (approximately 0.016 molar) is poured into this form, its free surface intersects the glass along a line representing the center line of the transition, while its intersection with the Lucite boundary represents the juncture of the conduit sides. Although the radius R of the cylindrical section (Fig. 2) remains fixed, its ratio to the conduit width B may be varied at will over a wide range simply by adjusting the elevation of the free surface. In this manner a single inexpensive form is used to represent an infinite number of circular transitions with varied curvature. This extreme flexibility is possibly the most important advantage the analog has over other methods.

Results.—The results of measurements on this model are summarized here. In Fig. 4, isobars (or isovels) estimated from the potential measurements have been drawn, together with curves showing the pressure distributions along the center of a side and along the juncture between two sides, for one

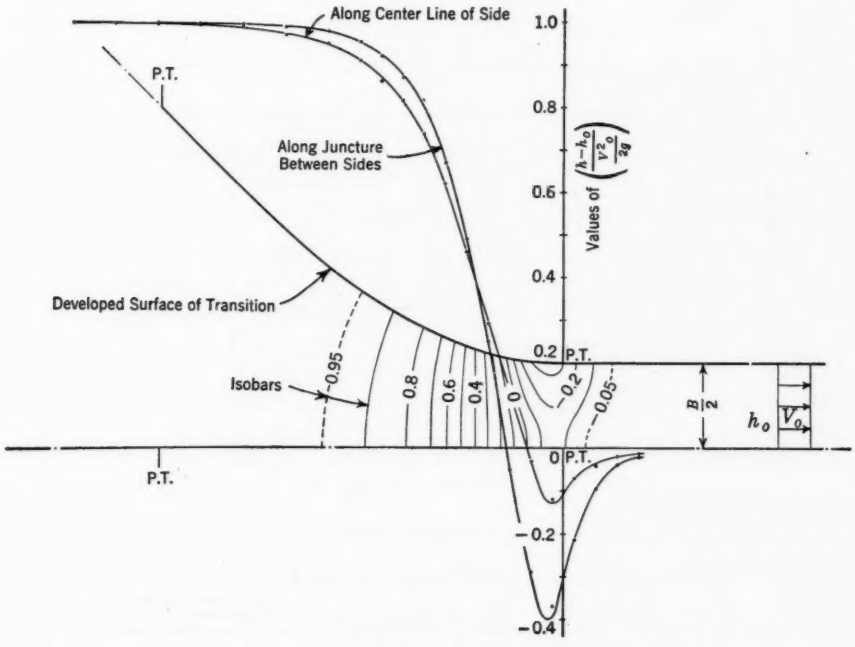

FIG. 4.—PATTERN OF PRESSURES ALONG BOUNDARY OF ROUNDED TRANSITION FROM A RESERVOIR TO A SQUARE CONDUIT $\left(\frac{R}{B} = 1.5\right)$

particular relative radius of curvature. The curves show quite clearly that the most critical region is along the juncture, just upstream from the point at which the transition curve becomes tangent to the straight conduit. However, pressures below that in the conduit prevail in a small region completely across the side at this zone. It should be noted again that the measurements are taken only at the boundary (where the most extreme conditions always occur) and that pressures away from the boundary will behave differently. In particular, it should be expected that the pressure along the center line of the conduit will decrease uniformly from the reservoir to the conduit with no underpressure. For prediction of cavitation or over-all forces, however, only boundary pressures are of interest.

In Fig. 5, a family of curves shows the variation in pressure distribution along the juncture (which includes the most critical point) for various relative radii of curvature. This figure indicates quantitatively the benefits achieved from decreasing the curvature. The trend with decreasing curvature is indicated more clearly by the curves of Fig. 6, in which the minimum pressures

along the juncture and along the center line of a side are plotted against the relative radius of curvature.

As an aid in applying the results of these tests, it may be noted that the values of $\frac{(h - h_0)}{\frac{V^2_0}{(2\,g)}}$ presented in this paper probably represent actual condi-

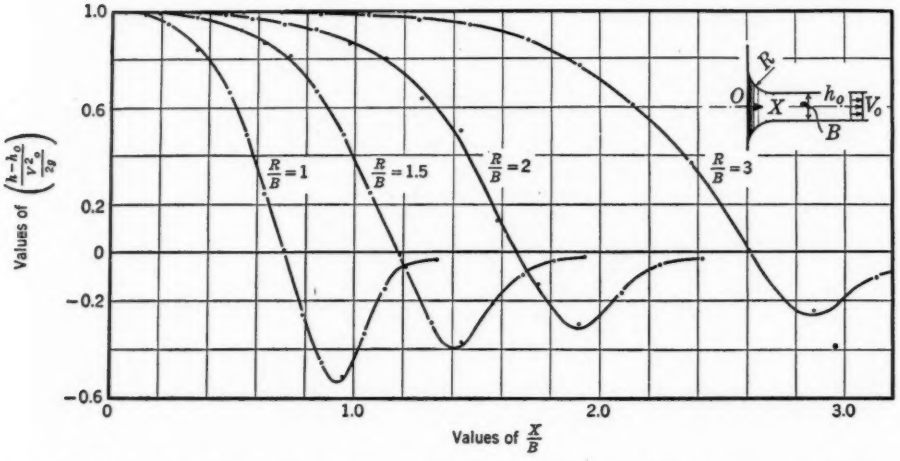

FIG. 5.—EFFECT OF RELATIVE CURVATURE ON PRESSURE DISTRIBUTION ALONG JUNCTURE OF SIDES

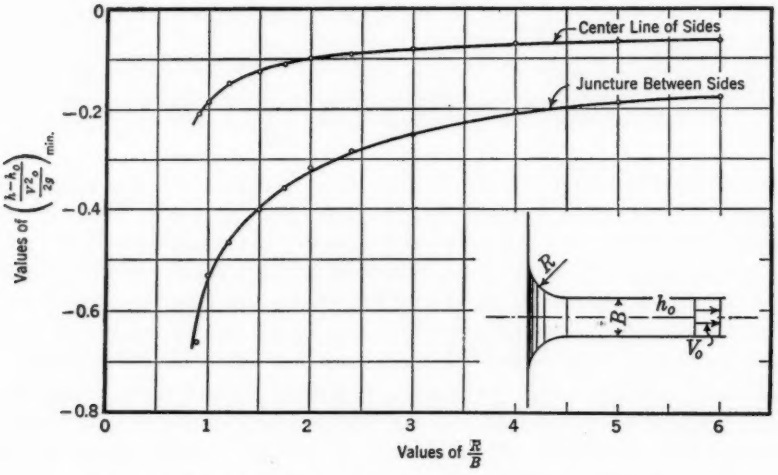

FIG. 6.—MINIMUM PRESSURE AS A FUNCTION OF RELATIVE CURVATURE

tions within a few percent, except where local deceleration will lead to separation. Since, contrary to the conclusions of other research workers,[5] the tendency toward cavitation decreases indefinitely with increasing radius of curvature, the design radius should be made as large as is consistent with con-

struction requirements. Other factors will influence the value chosen, of course, such as slope, roughness and length of the conduit, and the hydrostatic pressure at the outlet. Downward slopes, smooth surfaces, and short lengths of conduit will increase the easement of curvature necessary to achieve a given condition. The turbulent boundary layer of the prototype will decrease the tendency toward separation, but will not appreciably influence the cavitation characteristics predicted by the curves. The previous tests[5] correctly indicate that, for a given length of transition, underpressure may be minimized more effectively by using sections from elliptical—rather than circular—cylinders; the amount of improvement possible is still to be determined.

Conclusions

In solving problems such as the one discussed in this paper, the electrical analogy has proved to be a very valuable aid. Practically all the work of building a model can be completed without the aid of machine tools, and a single construction may simulate a wide range of boundary conditions if properly designed. Although the degree to which actual flow conditions approximate the potential pattern may be a subject of disagreement, there is no doubt that the electrical analogy permits a great deal of preliminary design work to be done economically. Then, if desired, the selected design may be constructed and tested under actual flow conditions.

Acknowledgment

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